Postulates of Quantum Mechanics

Postulate 1: The State of a Quantum System

Statement: The state of a quantum system is completely described by a **state vector** |ψ⟩ (wave function) in a **Hilbert space**. The state must be **normalized**:

$$\langle\psi|\psi\rangle = 1$$

Meaning: A quantum system (like a qubit, electron, or photon) is represented by a vector of complex probability amplitudes.

Example (Qubit): A qubit is described as:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where $|\alpha|^2 + |\beta|^2 = 1$

If $\alpha = \frac{1}{\sqrt{2}}$ and $\beta = \frac{1}{\sqrt{2}}$:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

This is a **superposition**.

Postulate 2: Time Evolution of a Quantum System

Statement: The time evolution of a closed quantum system is **unitary**, and it is governed by the **Schrödinger equation**:

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$

Meaning: Quantum states change smoothly over time using unitary operators (like quantum gates).

Example (Hadamard Gate): Applying the Hadamard gate H to |0⟩ :

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

This changes the state but keeps probability = 1.

Postulate 3: Measurement Postulate

Statement: Each observable is represented by a **Hermitian operator** $\hat{A}$ When measured, the result is one of its **eigenvalues**, and the system collapses to the corresponding **eigenstate**.

Probability of outcome a_i:

$$P(a_i) = |\langle a_i|\psi\rangle|^2$$

Meaning: Measurement gives definite outcomes, but the result is **probabilistic**, not deterministic. After measurement, the wave function **collapses**.

Example (Measuring a Qubit):

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Measuring in the computational basis:

- Probability of $|0\rangle = \frac{1}{2}$
- Probability of $|1\rangle = \frac{1}{2}$

After measurement, the state becomes either $|0\rangle$ or $|1\rangle$ — **not superposition anymore**.

Postulate 4: Expectation Values

Statement: The expectation (average) value of an observable $\hat{A}$ in state $|\psi\rangle$ is:

$$\langle A\rangle = \langle\psi|\hat{A}|\psi\rangle$$

Meaning: This tells us what value we *expect on average* if we repeat the same experiment many times.

Example:

If $\hat{Z}$ is Pauli-Z operator:

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

For |0⟩:

$$\langle Z \rangle = \langle 0|Z|0 \rangle = 1$$

Postulate 5: Composite Systems (Tensor Product)

Statement: The state of a composite system is the **tensor product** of the states of its subsystems.

Meaning: To describe multiple qubits, we join their states using tensor products.

Example (Two Qubits):

$$|0\rangle \otimes |1\rangle = |01\rangle$$

This framework allows **entanglement**, such as the Bell state:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Summary Table

Postulate	Key Idea	Example
1. State	System ↦ vector in Hilbert space	Qubit superposition
2. Evolution	Unitary time evolution (Schrödinger equation)	Hadamard gate
3. Measurement	Probabilistic outcome, wavefunction collapse	Measure qubit
4. Expectation	Average value of observable	Pauli-Z
5. Composite Systems	Tensor product of subsystems	Entangled Bell pair