PAI_Module4

Topics:

1. First Order Logic:
   a. **Representation Revisited,**
   b. **Syntax and Semantics of First Order Logic,**
   c. **Using First Order Logic**.
2. Inference in First Order Logic:
   a. **Propositional Versus First Order Inference,**
   b. **Unification,**
   c. **Forward Chaining,**
   d. **Backward Chaining**
   e. **Resolution**

4.1 First Order Logic

Properties of PL

1. Propositional logic is a **declarative language** because its semantics is based on a truth relation between sentences and possible worlds.
2. It also has sufficient **expressive power to deal with partial information**, using disjunction and negation.
3. Propositional logic has a third property that is desirable in representation languages, namely, **compositionality.** In a compositional language, the meaning of a sentence is a function of the meaning of its parts.

For example, the meaning of “S1,4 ∧ S1,2” is related to the meanings of “S1,4” and “S1,2.”

Drawbacks of Propositional Logic

- **Propositional logic** (PL) is declarative and assumes the world contains facts, so it guides us on how to represent information *in a logical form* and draw conclusions.
- We can only represent information as either **true or false** in propositional logic.
- Expressive power of Propositional logic is very limited and lacks to describe an environment with many objects
- If you want to represent complicated sentences or natural language statements, PL is not sufficient.
- **Examples:** PL is not enough to represent the sentences below, so we require powerful logic (such as FOL).
    1. I love mankind. It’s the people I can’t stand!
    2. I like to eat mangos.

What is First Order Logic (FOL)?

1. FOL is also called *predicate logic*. A much more expressive language than the propositional logic. It is a powerful language used to develop information about an **object and express the relationship** between objects.
2. FOL not only assumes that does the world contains facts (like PL does), but it also assumes the following:
    1. **Objects**: A, B, people, numbers, colors, wars, theories, squares, pit, etc.
    2. **Relations**: It is unary relation such as red, round, sister of, brother of, etc.
    3. **Function**: father of, best friend, third inning of, end of, etc.

First Order Logic Sentences

For each of the following English sentences, write a corresponding sentence in FOL.

1. The only good extraterrestrial is a drunk extraterrestrial.
   $\forall x.ET(x) \land Good(x) \rightarrow Drunk(x)$
2. The Barber of Seville shaves all men who do not shave themselves.
   $\forall x.\neg Shaves(x, x) \rightarrow Shaves(BarberOfSeville, x)$
3. There are at least two mountains in England.
   $\exists x, y.Mountain(x) \land Mountain(y) \land InEngland(x) \land InEngland(y) \land x \neq y$
4. There is exactly one coin in the box.
   $\exists x.Coin(x) \land InBox(x) \land \forall y.(Coin(y) \land InBox(y) \rightarrow x = y)$
5. There are exactly two coins in the box.
   $\exists x, y.Coin(x) \land InBox(x) \land Coin(y) \land InBox(y) \land x \neq y \land \forall z.(Coin(z) \land InBox(z) \rightarrow (x = z \lor y = z))$
6. The largest coin in the box is a quarter.
   $\exists x.Coin(x) \land InBox(x) \land Quarter(x) \land \forall y.(Coin(y) \land InBox(y) \land \neg Quarter(y) \rightarrow Smaller(y, x))$
7. No mountain is higher than itself.
   $\forall x.Mountain(x) \rightarrow \neg Higher(x, x)$
8. All students get good grades if they study.
   $\forall x.Student(x) \land Study(x) \rightarrow GetGoodGrade(x)$

Objects Relations and Functions

- **Objects:** people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries ...
- **Relations**: these can be unary relations or properties such as red, round, bogus, prime, multistoried ..., or more general n-ary relations such as brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, ...
- **Functions**: father of, best friend, third inning of, one more than, beginning of ..

Examples

- "One plus two equals three."
  - **Objects**: one, two, three, one plus two;
  - **Relation**: equals;
  - **Function**: plus. ("One plus two" is a name for the object that is obtained by applying the function "plus" to the objects "one" and "two." "Three" is another name for this object.)
- "Squares neighboring the wumpus are smelly."
  - **Objects**: wumpus, squares;
  - **Property**: smelly; Relation: neighboring.
- "Evil King John ruled England in 1200."
  - **Objects:** John, England, 1200;
  - **Relation**: ruled;
  - **Properties**: evil, king.

Types of Languages

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

Basic Elements of FOL

Constant	1, 2, A, John, Mumbai, cat,....
Variables	x, y, z, a, b,....
Predicates	Brother, Father, >, <,Sister, Father.......
Function	sqrt, LeftLegOf, Sqrt, LessThan, Sin(θ).......
Connectives	∧, ∨, ¬, ⇒, ⇔
Equality	==
Quantifier	∀, ∃

Syntax and Semantics of FOL

1. Models for first-order logic
2. Symbols and interpretations
3. Terms
4. Atomic sentences
5. Complex sentences
6. Quantifiers: Universal quantification (∀) /Existential quantification (∃)
7. Equality
8. An alternative semantics? : Data base Semantics

Models for FOL

- They have objects in them!
- The domain of a model is the set of objects or domain elements it contains.
- The domain is required to be nonempty—every possible world must contain at least one object
- The objects in the model may be related in various ways.
- Models in first-order logic require total functions, that is, there must be a value for every input tuple

Example : Consider the figure below which illustrates the model containing five objects, two binary relations, three unary relations and one unary functions.

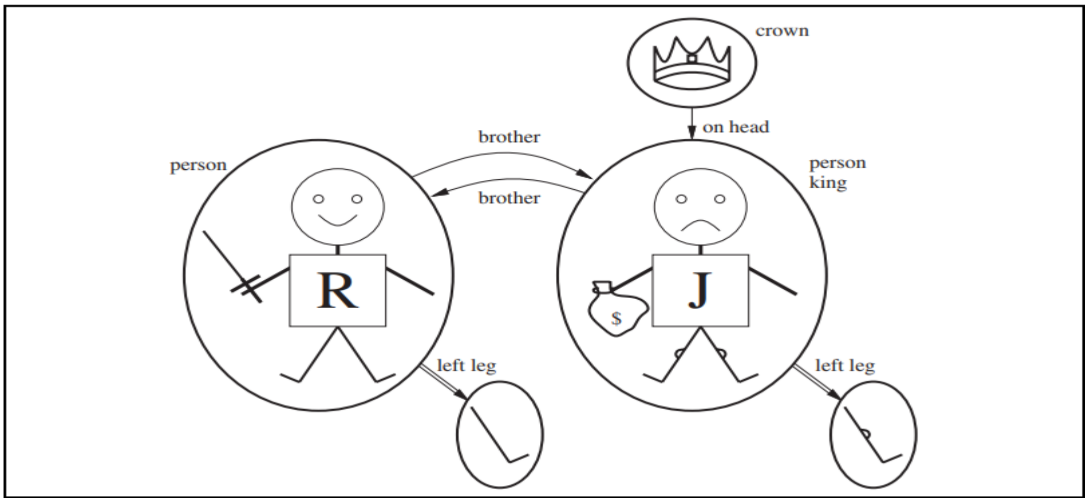

Figure 8.2 A model containing five objects, two binary relations, three unary relations (indicated by labels on the objects), and one unary function, left-leg.

Five objects:

1. Richard the Lionheart, King of England from 1189 to 1199;
2. His younger brother, the evil King John, who ruled from 1199 to 1215;
3. The left legs of Richard and John; and a c
4. Crown

Tuple:

The brotherhood relation in this model is the set { <Richard the Lionheart, King John>, <King John, Richard the Lionheart> } .

Two binary relations:

"brother" and "on head" relations are binary relations

Three unary relations/ properties:

Person, King and Crown

One unary Function:

Left Leg

Syntax of FOL

The basic syntactic elements of first-order logic are the symbols that stand for objects, relations, and functions. The symbols, therefore, come in three kinds:

1. **Constant symbols**, which stand for objects;
2. **Predicate symbols**, which stand for relations; and
3. **Function symbols**, which stand for functions.

Convention: Symbols will begin with uppercase letters.

Example

1. Constant symbols Richard and John;
2. Predicate symbols Brother , OnHead, Person, King, and Crown; and
3. the function symbol LeftLeg.

Arity:

Each predicate and function symbol comes with an arity that fixes the number of arguments

Interpretation:

Interpretation specifies exactly which objects, relations and functions are referred to by the constant, predicate, and function symbols.

Examples:

- **Richard** refers to Richard the Lionheart
- **John** refers to the evil King John.
- **Brother** refers to the brotherhood relation
- **OnHead** refers to the “on head” relation that holds between the crown and King John;
- **Person, King, and Crown** refer to the sets of objects that are persons, kings, and crowns
- **LeftLeg** refers to the “left leg” function

The syntax of first-order logic with equality, specified in Backus–Naur form

$$
\begin{array}{rcl}
Sentence & \rightarrow & AtomicSentence \mid ComplexSentence \\
AtomicSentence & \rightarrow & Predicate \mid Predicate(Term, \ldots) \mid Term = Term \\
ComplexSentence & \rightarrow & (\ Sentence\ ) \mid [\ Sentence\ ] \\
 & \mid & \neg\ Sentence \\
 & \mid & Sentence \land Sentence \\
 & \mid & Sentence \lor Sentence \\
 & \mid & Sentence \Rightarrow Sentence \\
 & \mid & Sentence \Leftrightarrow Sentence \\
 & \mid & Quantifier\ Variable, \ldots\ Sentence \\
 & & \\
Term & \rightarrow & Function(Term, \ldots) \\
 & \mid & Constant \\
 & \mid & Variable \\
 & & \\
Quantifier & \rightarrow & \forall \mid \exists \\
Constant & \rightarrow & A \mid X_1 \mid John \mid \cdots \\
Variable & \rightarrow & a \mid x \mid s \mid \cdots \\
Predicate & \rightarrow & True \mid False \mid After \mid Loves \mid Raining \mid \cdots \\
Function & \rightarrow & Mother \mid LeftLeg \mid \cdots \\
\textsc{Operator Precedence} & : & \neg, =, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

In summary

- A model in first-order logic consists of a set of objects and an interpretation that maps constant symbols to objects, predicate symbols to relations on those objects, and function symbols to functions on those objects.
- Just as with propositional logic, entailment, validity, and so on are defined in terms of all possible models